



# Cambridge International AS & A Level

CANDIDATE  
NAME

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## FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

October/November 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

- 1 (a) By differentiating  $e^{-x^2}$ , find the Maclaurin's series for  $e^{-x^2}$  up to and including the term in  $x^2$ . [5]

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- (b) Deduce an approximation to  $\int_0^{\frac{1}{5}} e^{-x^2} dx$ , giving your answer as a rational fraction in its lowest terms. [2]

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2 The variables  $x$  and  $y$  are related by the differential equation

$$9 \frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + y = 3x^2 + 30x.$$

(a) Find the general solution for  $y$  in terms of  $x$ . [6]

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(b) State an approximate solution for large positive values of  $x$ . [1]

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3 (a) Show that the system of equations

$$\begin{aligned}x - 2y - 4z &= 1, \\x - 2y + kz &= 1, \\-x + 2y + 2z &= 1,\end{aligned}$$

where  $k$  is a constant, does not have a unique solution. [2]

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(b) Given that  $k = -4$ , show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]

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- (c) Given instead that  $k = -2$ , show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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- (d) For the case where  $k \neq -2$  and  $k \neq -4$ , show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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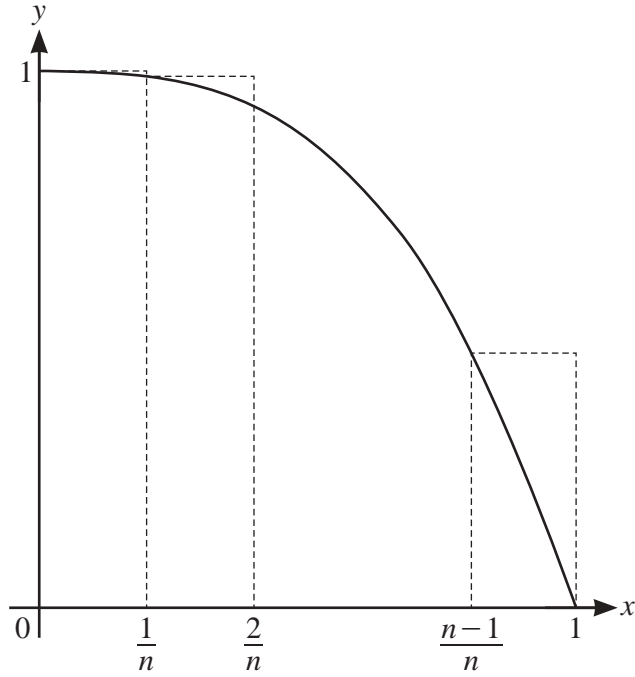
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The diagram shows the curve with equation  $y = 1 - x^3$  for  $0 \leq x \leq 1$ , together with a set of  $n$  rectangles of width  $\frac{1}{n}$ .

(a) By considering the sum of the areas of the rectangles, show that

$$\int_0^1 (1 - x^3) dx \leq \frac{3n^2 + 2n - 1}{4n^2}. \quad [4]$$

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(b) Find  $\frac{d^2y}{dx^2}$  in terms of  $t$ , simplifying your answer.

[5]

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6 (a) Use de Moivre's theorem to show that  $\sin^4 \theta = \frac{1}{8}(\cos 4\theta - 4 \cos 2\theta + 3)$ . [5]

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7 The matrix  $\mathbf{P}$  is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

- (a) State the eigenvalues of  $\mathbf{P}$ . [1]

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- (b) Use the characteristic equation of  $\mathbf{P}$  to find  $\mathbf{P}^{-1}$ . [4]

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The  $3 \times 3$  matrix  $\mathbf{A}$  has distinct eigenvalues  $b, -1, 1$  with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix},$$

respectively.

(c) Find  $\mathbf{A}$  in terms of  $b$ .

[4]

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8 (a) Sketch the graph of  $y = \coth x$  for  $x > 0$  and state the equations of the asymptotes. [2]

(b) Starting from the definitions of  $\coth$  and  $\operatorname{cosech}$  in terms of exponentials, prove that

$$\coth^2 x - \operatorname{cosech}^2 x = 1. \quad [3]$$

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**Additional Page**

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